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EFFECT OF THE NONMONOTONIC TEMPERATURE DEPENDENCE OF WATER DENSITY ON THE PROPAGATION OF A VERTICAL PLANE JET

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This paper gives results of two experiments in which warmer water in the form of a vertical plane jet was let into colder water. In one experiment, the temperature of inflowing and initially quiescent water exceeded 4°C *and warm water propagated only along the free surface. In the other experiment, the temperature of inflowing water was above* 4°C *and that of quiescent water was below* 4°C*. In this case, two jets — surface and bottom — first formed, and then the inflowing liquid was entirely concentrated in the bottom jet.*

Key words: *temperature of maximum density water, Rayleigh–Taylor instability, surface jet, bottom jet.*

At atmospheric pressure, the density of distilled water is maximal at a temperature T_* of about 4°C, and at a temperature of 0°C it is approximately 0.07% lower. When water masses at temperatures $T_1 > T_*$ and $T_2 < T_*$ come in contact, heat transfer results in the formation of a maximum density layer, which is unstable by the Rayleigh–Taylor mechanism. Below, the hydrodynamic processes due to the nonmonotonic temperature dependence of water density will be called for brevity anomalous effects. Conditions for the occurrence of such effects are produced in fresh water pools as a result of ice melting, fluvial runoff, rainfall and snowfall, and household and process water discharges. The difference in temperature between the shallow- and deep-water parts of a pool leads to the formation of vernal and autumnal thermal bars [1–3]. Under laboratory conditions, the anomalous effects have been studied in the cases of propagation of plane and circular horizontal jets [4–6]. The present paper gives results of experiments shown schematically in Fig. 1.

A tank 3.8 m long and 20 cm wide was divided into two parts by a vertical partition with a slot below. In the initial state, the slot was covered with a shield. The left (in the diagram) part of the tank was filled with a weak aqueous solution colored with ink at temperature $T_1 > T_*$. Water in the right part at temperature $T_2 < T_1$ was clear. Below, we give results of two experiments. In experiment No. 1, $T_1 = 20.7$°C and $T_2 = 10.7$°C. Both these values are larger than T_*; therefore, contact of colored and clear water produced no anomalous effects. In experiment No. 2, $T_1 = 19.7$°C and $T_2 = 0.2$°C, so that conditions for the occurrence of anomalous effects were available. In both experiments, the initial density difference on the left and right of the partition was identical — 0.159% of the maximum water density. The processes were recorded by a video camera with a frequency 25 frames/sec and were analyzed on a computer.

Between the right and left parts of the tank, an initial level difference was produced, so that after the lift of the shield, lighter water in the form of a plane jet flowed into heavier water. The vertical barrier (see Fig. 1) deflected the jet upward. The jet rise was also facilitated by the initial, upward directed, buoyancy force (the difference between the gravity and Archimedean force). In both experiments, the free surface levels on the left and right of the partition became almost equal in approximately 60–65 sec. After that, the additional supply of the

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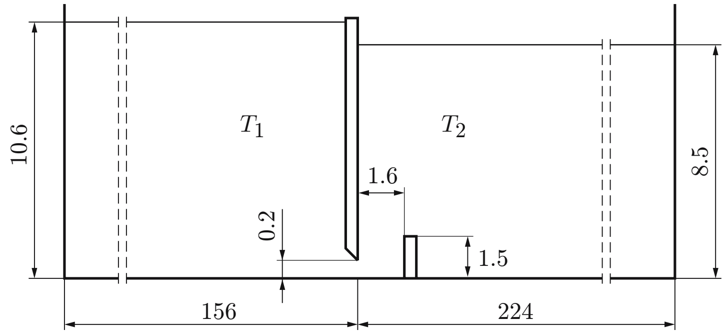

Fig. 1. Diagram of experiment (the dimensions are in centimeters).

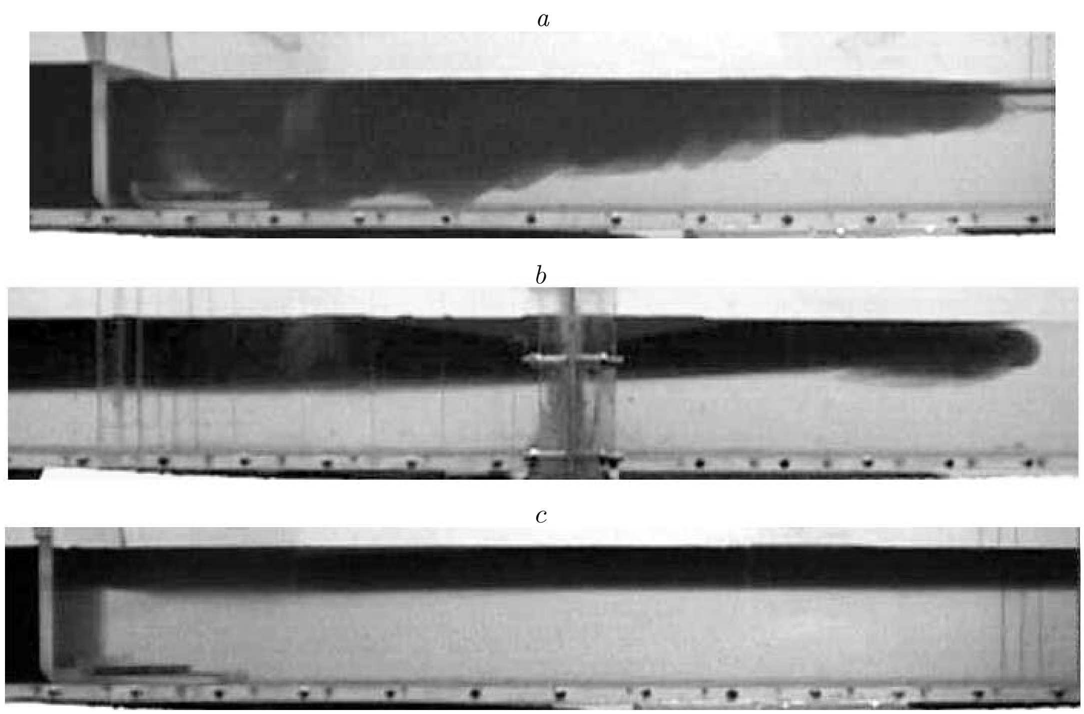

Fig. 2. Flow pattern in experiment No. 1 at $t = 38$ (a), 128 (b), and 325 sec (c).

colored liquid to the right side of the tank ceased and the processes occurred only due to inertia and buoyancy forces.

Figure 2 gives three frames of video experiment No. 1 taken at various times t from the beginning of the lift of the shield. In this experiment, anomalous effects were absent since the colored liquid propagated in the form of a plane jet only along the free surface, as in other previous experiments (see, for example, [7]). For approximately 60 sec there was mixing of the warm and cold water because of Kelvin–Helmholtz shear instability (Fig. 2a). Subsequently, a characteristic bulge of the jet head formed and weak mixing continued only on a small segment behind this bulge (Fig. 2b). After the reflection of the jet from the butt-end wall of the tank, the mixing completely ceased and only weak smooth waves (Fig. 2c) propagated over the interface between warm and cold water.

Figure 3 shows several frames taken in experiment No. 2. Because of anomalous effects, the flow pattern in this experiment differed radically from that given in Fig. 2, starting with the earliest stage of intrusion of colored water into transparent water. Within the first 10–12 sec, colored and transparent waters were separated by a vertical front, as is the case during the formation of a thermal bar. Then, because of turbulent mixing, the part of water in the neighborhood of the thermal bar gained density close to the maximum and sank to the bottom, and

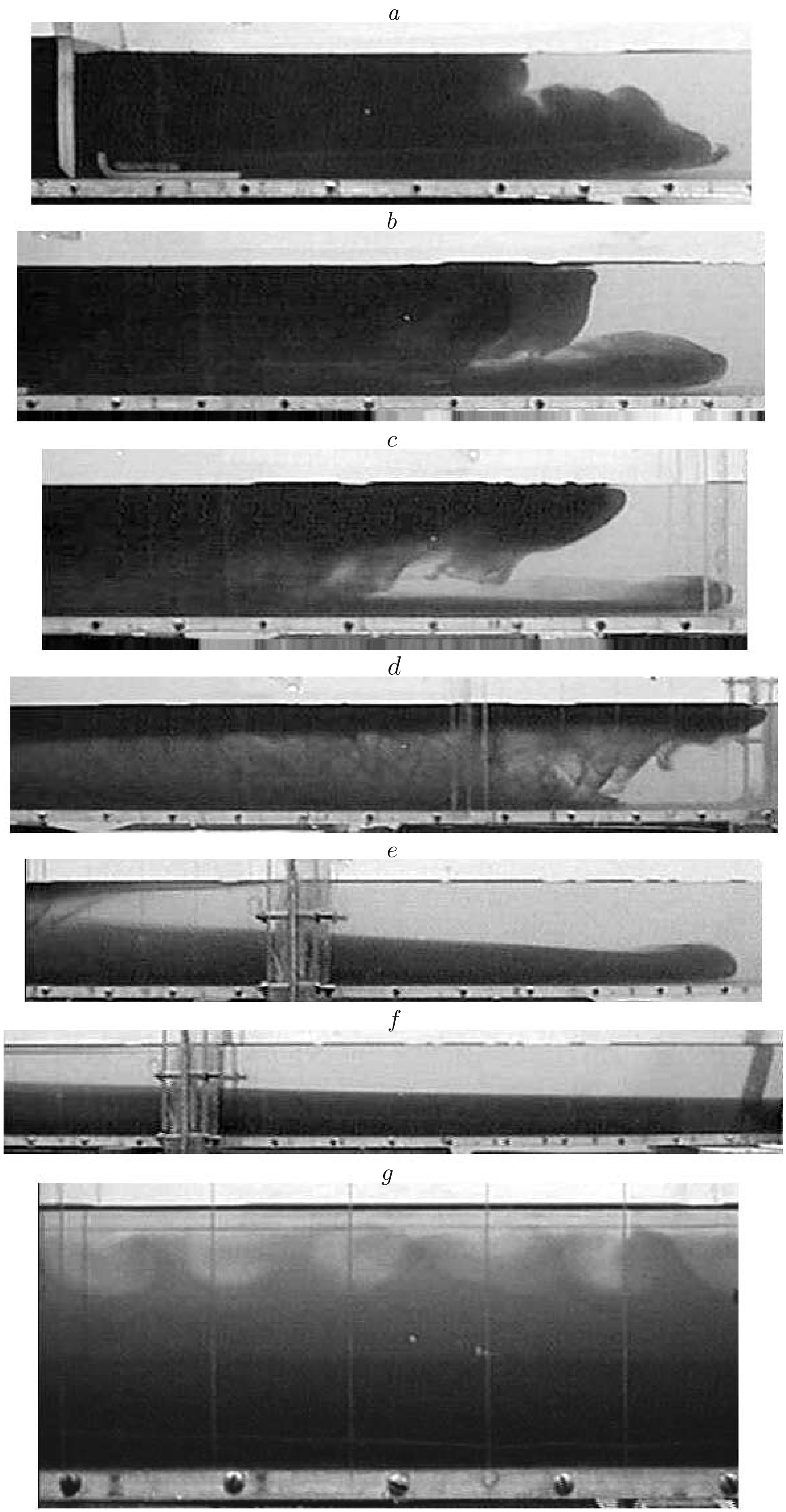

Fig. 3. Flow pattern in experiment No. 2 at $t = 42$ (a), 70 (b), 100 (c), and 199 (d), 368 (e), 756 (f), and 3900 sec (g).

most of the mass of inflowing warmer water propagated in the form of a surface jet with an almost vertical leading edge (Fig. 3a and b). Subsequently, water from the upper jet intensely sank to the lower jet because of instability of the maximum density layer resulting from the mixing and heat transfer at the lower boundary of the upper jet (Fig. 3c and d).

In a certain stage of the process in experiment No. 2, the leading edge of the surface jet began to move backward and the colored liquid appeared at the bottom of the tank (Fig. 3e and f). In approximately 15 min, the motion visible to the naked eye stopped. Within further approximately 20 min, stable stratification with temperatures of 4.5–5.0°C in the colored bottom layer and 0.4–0.5°C in the surface layer persisted. After that, the less dense clear water began to sink and the heavier gravity colored water began to rise against buoyancy forces (Fig. 3g).

There are various factors responsible for the motion of an initially quiescent, stably density-stratified fluid against buoyancy forces. Much attention has been paid to the hydrophysical effects resulting from the considerable difference between the quantitative values of the molecular diffusivity of heat and dissolved salt. These effects are described using the term double diffusion [8, 9]. In the experiments discussed here, the role of dissolved salt was played by ink, whose molecular diffusivity is approximately 100 times lower than that of heat. Double diffusion effects are especially pronounced if the signs of the temperature and salt concentration gradients are unlike [8]. In the experiments discussed here, they were of equal sign.

One more possible cause of the motion against buoyancy forces is a memory of the initial vorticity [6]. With time, large vortices due to the initial vorticity break up into a set of small-size vortices. In a stably stratified liquid, the axis of the small-size vortices predominantly points up. At the center of the vortices there is a low-pressure zone. Liquid can be entrained in this zone even against a considerable buoyancy force. A vivid example is the formation of an air funnel during emptying of a bath, where water entrains air, whose density is approximately 800 times lower than the density of water.

The most probable cause of the discussed effect in the experiments considered is heat exchange with the laboratory room. The experiments were conducted at a room air temperature of 14.2°C. The tank was manufactured of Plexiglas 18 mm thick; therefore, heat transfer through the bottom and walls of the tank was insignificant. The determining factor was heat transfer through the free surface. In experiment No. 2, because the water temperature in the upper layer was below T_* while the room temperature was above T_*, a maximum density layer formed at the free surface; the submersion of this layer was accompanied by displacement of the colored liquid against buoyancy forces (Fig. 3g). This mechanism operated at all stages of the process. However, the effects due to this mechanism were observed most clearly after the termination of shear flow.

A similar mechanism also acted in experiment No. 1. In this experiment, an increase in density in a thin layer near the free surface was due primarily to water evaporation and not to the anomalous temperature dependence of water density and the rate of motion against buoyancy forces was much lower than that in experiment No. 2.

Figure 4 gives curves of x_i ($i = 0, 1, 2$) versus time t, which have the following physical meaning: x_0 is the longitudinal coordinate of the leading edge of the surface jet in experiment No. 1 and x_1 and x_2 are the longitudinal coordinates of the leading edges of the surface and bottom jets, respectively, in experiment No. 2. In the absence of anomalous effects, the coordinate of the leading edge of the jet increases monotonically (curve 1 in Fig. 4). In the presence of anomalous effects, curves 2 and 3 have plateaus, indicating that the leading edges of the jets stopped for a time and then started moving again in the positive x direction. In addition, curve 2 has a maximum. At the time corresponding to this maximum, the leading edge of the surface jet stopped again and began to move in the opposite direction.

In experiment No. 2, the first stop of the leading edge of the surface jet occurred in the stage of the process where energy supply to the jet due to the level difference between the left and right parts of the tank still continued. At the same time, the mechanism of formation of the maximum density front opposed the propagation of the jet. The orientation of this front was first predominantly vertical. Then, the length of the maximum density front in the vertical direction began to decrease rapidly, and the surface jet acquired positive acceleration. However, because of Rayleigh–Taylor instability, the surface jet continued to lose mass and energy, resulting in a secondary stop and backward motion of the colored liquid in the upper layer.

The motion of the colored water in the positive x direction generated reverse motion of transparent water. This also opposed the propagation of the surface and bottom jets and was accompanied, in particular, by charac-

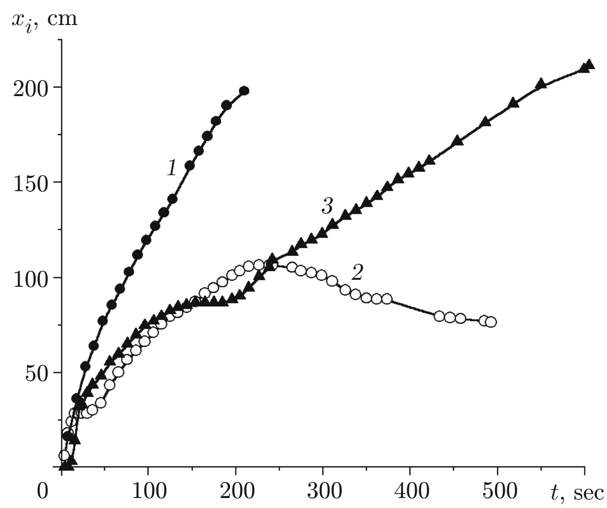

Fig. 4. Longitudinal coordinates of the leading edges of the jets versus time: curves 1, 2, and 3 refer to x_0, x_1, and x_2, respectively.

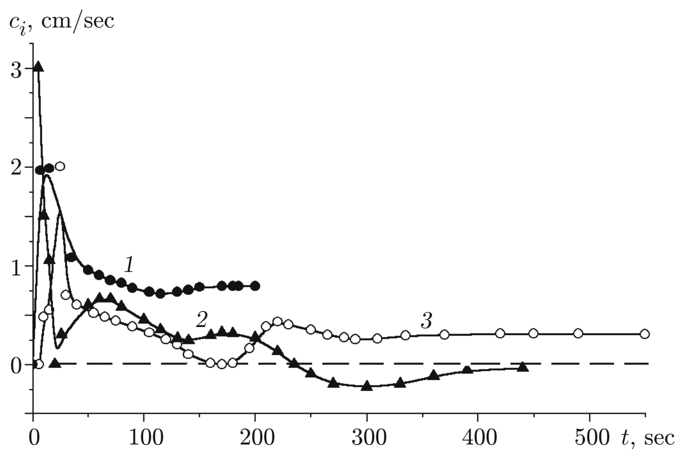

Fig. 5. Propagation speeds of the leading edges of the jets versus time: curves 1, 2, and 3 refer to c_0, c_1, and c_2, respectively.

teristic bulging of their heads (see Fig. 3). In a certain stage of the process, this opposing mechanism led to the fact that the leading edge of the lower layer also stopped, and a plateau appeared in curve 3 (see Fig. 4); it appeared much later than that in curve 2. Subsequently, however, the mass and potential energy of the lower layer increased so that its leading edge start moving again in the positive x direction.

Figure 5 gives curves of the propagation speeds of the leading edges of the jets c_i $(i = 0, 1, 2)$ versus time t. Subscripts $i = 0$ refers to the surface jet in experiment No. 1 and subscripts $i = 1$ and 2 refer to the surface and bottom jets, respectively, in experiment No. 2. Only the time period before the reflection of the jets from the butt-end wall of the tank is considered. From Fig. 5 it is evident that except in a short initial time interval, the speed c_0 far exceeds the speeds c_1 and c_2. After the free surface levels in the left and right parts of the tank had become identical, the quantity c_0 rapidly reached a constant value of $c_0 \approx 0.79$ cm/sec at $t > 150$ sec.

At $t \approx 230$ sec, the propagation speed of the leading edge of the surface jet c_1 in experiment No. 2 entered the region of negative values and began to tend to zero asymptotically. At $t > 350$ sec, the propagation speed of the leading edge of the bottom jet took a constant value $c_2 \approx 0.31$ cm/sec.

In conclusion, we note that the effects considered here can be of significance during water discharge from thermal or atomic power plants into a pool in spring and autumn, and also in convection processes in deep pools such as lake Baikal. In the first case, the discharged water, instead of propagating along the free surface, can sink to the bottom. In lakes, the temperature of maximum density water is preserved at a depth of a few hundred meters from the free surface throughout the year. At the same time, at the bottom there is constant heating and an unstable layer of lighter water forms. Because of the anomalous temperature dependence of density, heated water can rise only to a limited height. The thermal energy supplied from the lake bottom becomes potential energy, which is a peculiar example of operation of a heat engine in nature. It should be noted that the temperature of maximum density water depends on the pressure and decreases with depth.

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